Journal of Engineering Physics and Thermophysics, Vol. 79, No. 3, 2006

THERMOACTIVE COATING AS A MEANS FOR CONTROLLED ACTION ON THE TEMPERATURE FIELD OF AN INFINITE SOLID BODY WITH A SPHERICAL HEATING SOURCE

A. V. Attetkov UDC 536.2

A mathematical model of the process of formation of a temperature field in an infinite isotropic solid body containing a spherical heating source with a thermally thin thermoactive coating of its surface has been proposed. The obtained analytical solution of the corresponding problem of nonstationary heat conduction has been used for substantiation of the possibility of acting on the temperature field of the system under study in a controlled manner.

Investigations associated with a mathematical model of a spherical heating source and with studying the processes of heat and mass transfer in the solid body–gas system [4–10] occupy a highly important place in the applications of mathematical heat-conduction theory [1–3]. Despite the large number of publications, it is unlikely that investigations on this problem may be considered as being completed. In particular, it remains topical to find the temperature field in the system under study in the presence of a thermally thin coating on the surface of a spherical heating source. This problem is of special interest in investigations of the shock-wave sensitivity of energy materials [11–13]. It involves substantiation of the possibility of acting on the temperature field of the system under study in a controlled manner. One can attain this, for example, by surface phlegmatization of the heating source with the use of both chemically inert low-strength and easily flowing (fluid) additions and those chemically active [13–15]. A coating formed in the process of phlegmatization may be considered as being thermally thin, as a rule, because of its small thickness.

The main objective of the investigations carried out is to study the distinctive features of the process of formation of a temperature field in an infinite isotropic solid body containing a spherical heat source with a thermoactive (heat-absorbing) thermally thin coating of its surface.

Formulation of the Problem and a Mathematical Model. Let us consider an infinite isotropic solid body containing a spherical heating source (spherical cavity filled with high-temperature gas) with a thermoactive coating of its surface; the specific heat-absorption power in it is equal to q (Fig. 1). We will assume that thermal contact in the solid body–coating system is ideal and heat exchange with the gas filling the spherical cavity follows the Newton law with a constant heat-transfer coefficient α [2, 3].

Under the assumptions made and in the presence of a central symmetry, the initial model of the process of formation of a temperature field in an infinite isotropic solid body containing a spherical heating source with a thermoactive coating of its surface may be represented in the following form:

$$\frac{\partial \Theta}{\partial \mathrm{Fo}} = \frac{1}{\rho^2}\frac{\partial}{\partial \rho}\left(\rho^2 \frac{\partial \Theta}{\partial \rho}\right), \quad \rho > R > 1\,, \quad \mathrm{Fo} > 0\,;$$

$$\frac{\partial \Theta_c}{\partial \mathrm{Fo}} = \chi\left[\frac{1}{\rho^2}\frac{\partial}{\partial \rho}\left(\rho^2 \frac{\partial \Theta_c}{\partial \rho}\right) - \Lambda f(\rho, \mathrm{Fo})\right], \quad 1 < \rho < R\,, \quad \mathrm{Fo} > 0\,;$$

$$\Theta(\rho, \mathrm{Fo})\Big|_{\mathrm{Fo}=0} = 0 = \Theta_c(\rho, \mathrm{Fo})\Big|_{\mathrm{Fo}=0}\,; \qquad (1)$$

Scientific-Research Institute of Special Mechanical Engineering, 10 Gospitalnyi Pereulok, Moscow, 105005, Russia; email: apmath@bmstu.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 79, No. 3, pp. 12–19, May–June, 2006. Original article submitted November 30, 2004.

1062-0125/06/7903-0429

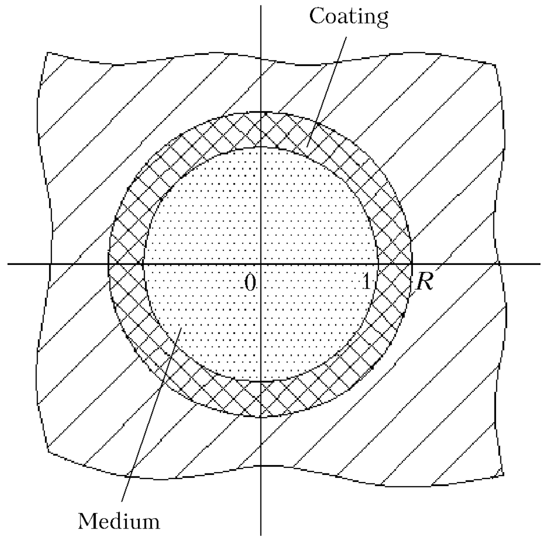

Fig. 1. Calculated diagram for studying the process of formation of a temperature field in a solid body containing a spherical heating source with a thermally thin thermoactive coating of its surface.

$$\left.\frac{\partial \Theta_c(\rho, \mathrm{Fo})}{\partial \rho}\right|_{\rho=1} = \Lambda\, \mathrm{Bi}\, [\Theta_c(\rho, \mathrm{Fo})\big|_{\rho=1} - \zeta(\mathrm{Fo})] ;$$

$$\Theta_c(\rho, \mathrm{Fo})\Big|_{\rho=R-0} = \Theta(\rho, \mathrm{Fo})\Big|_{\rho=R+0} , \quad \left.\frac{\partial \Theta_c(\rho, \mathrm{Fo})}{\partial \rho}\right|_{\rho=R-0} = \Lambda \left.\frac{\partial \Theta(\rho, \mathrm{Fo})}{\partial \rho}\right|_{\rho=R+0} ;$$

$$\rho\Theta(\rho, \mathrm{Fo})\Big|_{\mathrm{Fo}>0} \in L^2 [R, +\infty) ,$$

where the last condition means that the function $\Theta(\rho, \mathrm{Fo})$ is square-integrable in the space variable $\rho \cup [R, \infty]$ for each fixed $\mathrm{Fo} > 0$. In expressions (1), we have

$$\rho = \frac{r}{r_1}, \quad \mathrm{Fo} = \frac{at}{r_1^2}, \quad \Theta = \frac{T - T_0}{T_{m0} - T_0}, \quad \Theta_c = \frac{T_c - T_0}{T_{m0} - T_0}, \quad \zeta = \frac{T_m - T_0}{T_{m0} - T_0},$$

$$\chi = \frac{a_c}{a}, \quad \Lambda = \frac{\lambda}{\lambda_c}, \quad \mathrm{Bi} = \frac{\alpha}{\lambda} r_1 , \quad R = \frac{r_2}{r_1}, \quad f = \frac{q r_1^2}{\lambda (T_{m0} - T_0)} .$$

The assumption that the coating is thermally thin makes it possible to realize the idea of "lumped capacitance" [16, 17]. Introducing the coating thickness mean-integral over the thickness

$$\langle \Theta(\mathrm{Fo}) \rangle = \frac{3}{R^3 - 1} \int_1^R \Theta_c(\rho, \mathrm{Fo})\, \rho^2 d\rho$$

into consideration and making the assumption that the temperature of the coating boundaries is equal to both its mean-integral temperature and the temperature of the surface $\rho = R$ of a shielded spherical cavity, i.e.,

$$\Theta_c(1 + 0, \mathrm{Fo}) = \Theta_c(R - 0, \mathrm{Fo}) = \langle \Theta(\mathrm{Fo}) \rangle = \Theta(R + 0, \mathrm{Fo}) , \quad \mathrm{Fo} \geq 0 ,$$

we can simplify the initial mathematical model (1) and can transform it:

$$\frac{\partial\Theta}{\partial \mathrm{Fo}} = \frac{1}{\rho^2}\frac{\partial}{\partial\rho}\left(\rho^2\frac{\partial\Theta}{\partial\rho}\right), \quad \rho > R > 1 , \quad \mathrm{Fo} > 0 ; \qquad \Theta(\rho, \mathrm{Fo})\Big|_{\mathrm{Fo}=0} = 0 ;$$

$$R^2\frac{\partial\Theta(\rho, \mathrm{Fo})}{\partial\rho}\Bigg|_{\rho=R} = \mathrm{Bi}\left[\Theta(\rho, \mathrm{Fo})\big|_{\rho=R} - \zeta(\mathrm{Fo})\right] + \varepsilon\frac{\partial\Theta(\rho, \mathrm{Fo})}{\partial \mathrm{Fo}}\Bigg|_{\rho=R} + Q(\mathrm{Fo}) ; \tag{2}$$

$$\rho\Theta(\rho, \mathrm{Fo})\Big|_{\mathrm{Fo}>0} \in L^2[R, +\infty) ,$$

where $Q(\mathrm{Fo}) = \int_1^R f(\rho, \mathrm{Fo})\rho^2 d\rho$ is the integral quantity characterizing the heat-absorption regime realized in the thermally thin coating and $\varepsilon = (R^2 - 1)/(3\chi\Lambda)$ is the dimensionless governing parameter. We note that, according to the meaning of the problem being solved, the parameter ε takes on only positive values and is dependent on both the coating thickness and the relation of the thermophysical characteristics of the solid-body and coating materials.

The mathematical model (2) represents a mixed problem of nonstationary heat conduction, in which the presence of a thermally thin coating is allowed for by the boundary condition for $\rho = R$ explicitly containing a derivative of temperature with respect to time. When $\varepsilon = 0$ ($R = 1$), its solution has been obtained in [1] using the integral Laplace transformation in variable Fo. An analytical method of solution of problem (2) for $\varepsilon = 0$ for nonstationary regimes of heat exchange with the gas filling a spherical cavity has been proposed in [8]. The foundation of this method is the idea of splitting of the kernel of a mixed integral Fourier transform taken in the space variable.

To simplify further reasoning we introduce the function

$$V(\rho, \mathrm{Fo}) = \rho\Theta(\rho, \mathrm{Fo}) , \tag{3}$$

using a well-known technique [1]. Then the mathematical model (2) will be represented in the form

$$\frac{\partial V}{\partial \mathrm{Fo}} = \frac{\partial^2 V}{\partial\rho^2}, \quad \rho > R > 1 , \quad \mathrm{Fo} > 0 ;$$

$$V(\rho, \mathrm{Fo})\Big|_{\mathrm{Fo}=0} = 0 ;$$

$$R^2\frac{\partial V(\rho, \mathrm{Fo})}{\partial\rho}\Bigg|_{\rho=R} = (\mathrm{Bi} + R)\, V(\rho, \mathrm{Fo})\Big|_{\rho=R} - R\,\mathrm{Bi}\,\zeta(\mathrm{Fo}) + \varepsilon\frac{\partial V(\rho, \mathrm{Fo})}{\partial\rho}\Bigg|_{\rho=R} + RQ(\mathrm{Fo}) ; \tag{4}$$

$$V(\rho, \mathrm{Fo})\Big|_{\mathrm{Fo}>0} \in L^2[R, +\infty) .$$

Problem (4) has a unique solution [18], to find which we use the integral Laplace transformation in the variable Fo [1–3].

Temperature Field. Let the functions $V(\rho, \mathrm{Fo})$, $\zeta(\mathrm{Fo})$, and $Q(\mathrm{Fo})$ be the inverse transforms of the integral Laplace transformation,

$$L[\bullet] \equiv \int_0^\infty \exp(-s\mathrm{Fo})\bullet d\mathrm{Fo} \tag{5}$$

being the operator of the direct integral Laplace transformation in the variable Fo with a parameter $s \cup C$, and

$$U(\rho, s) = L[V(\rho, \mathrm{Fo})]\,, \quad \Psi(s) = L[\zeta(\mathrm{Fo})]\,, \quad \Pi(s) = L[Q(\mathrm{Fo})]\,. \tag{6}$$

Then, according to (4) and (6), the transform $U(\rho, s)$ of the integral Laplace transformation (5) of the function $V(\rho, \mathrm{Fo})$ must satisfy the equation

$$sU(\rho, s) = \frac{d^2 U(\rho, s)}{d\rho^2}\,, \quad \rho > R\,, \tag{7}$$

and the boundary condition

$$R^2 \frac{dU(\rho, s)}{d\rho} = [(\mathrm{Bi} + R) + \varepsilon s]\, U(\rho, s) - R\,[\mathrm{Bi}\, \Psi(s) - \Pi(s)]\,, \quad \rho = R\,, \tag{8}$$

and must belong to the class of $L^2[R, +\infty]$ functions quadratically integrable in the space variable $\rho \cup [R, +\infty]$ for each fixed value of the parameter s.

The solution $U(\rho, s)$ of Eq. (7) from the $L^2[R, +\infty]$ class has the following form:

$$U(\rho, s) = C(s) \exp(-\rho\sqrt{s})\,, \quad \rho \geq R\,, \tag{9}$$

where the function $C(s)$, according to (8) and (9), is determined as

$$C(s) = \frac{R\,[\mathrm{Bi}\, \Psi(s) - \Pi(s)]}{\varepsilon s + R^2 \sqrt{s} + (\mathrm{Bi} + R)} \exp(R\sqrt{s})\,. \tag{10}$$

To complete the procedure of solution of problem (4) we must also realize the passage from the transform $U(\rho, s)$ determined by equalities (9) and (10) to the inverse transform $V(\rho, \mathrm{Fo})$, which makes it possible, taking into account equality (3), to find the temperature field $\Theta(\rho, \mathrm{Fo})$ of the infinite solid body containing a spherical heating source with a thermally thin coating of its surface. Setting

$$\zeta(\mathrm{Fo}) = L^{-1}[\Psi(s)]\,, \quad Q(\mathrm{Fo}) = L^{-1}[\Pi(s)]\,, \tag{11}$$

where $L^{-1}[\bullet]$ is the operator of inversion of the integral Laplace transformation, taking into account equality (3) and the convolution theorem [2], we obtain

$$\Theta(\rho, \mathrm{Fo}) = \frac{R}{\rho} \int_0^{\infty} [\mathrm{Bi}\, \zeta(\mathrm{Fo} - \tau) - Q(\mathrm{Fo} - \tau)]\, \varphi(\rho, \tau)\, d\tau\,, \quad \rho \geq R\,, \quad \mathrm{Fo} \geq 0\,. \tag{12}$$

Here

$$\varphi(\rho, \mathrm{Fo}) = \frac{1}{\varepsilon} L^{-1}\left[\frac{\exp\left\{-(\rho - R)\sqrt{s}\right\}}{\left(\sqrt{s} + \dfrac{R^2}{2\varepsilon}\right)^2 + \left(\dfrac{\mathrm{Bi} + R}{\varepsilon} - \dfrac{R^4}{4\varepsilon^2}\right)}\right]. \tag{13}$$

Thus, if the form of the dependences $\zeta(\mathrm{Fo})$ and $Q(\mathrm{Fo})$ is specified, to determine the temperature field in the solid body we must only find the inverse transform $\varphi(\rho, \mathrm{Fo})$ of the corresponding transform. We note that, according to equality (13), there can be three representations of the function $\varphi(\rho, \mathrm{Fo})$ depending on the character of the roots of the quadratic equation $z^2 + \varepsilon^{-1}\,[R^2 z + (\mathrm{Bi} + R)] = 0$ [2, 19] in passage to the inverse transform:

$$\left(\mathrm{Bi} < \frac{R^4}{4\varepsilon} - R\right) \Rightarrow \varphi(\rho, \mathrm{Fo}) = \frac{1}{\varepsilon(\mu_2 - \mu_1)} L^{-1}\left[\left(\frac{1}{\sqrt{s} + \mu_1} - \frac{1}{\sqrt{s} + \mu_2}\right)\exp\left\{-(\rho - R)\sqrt{s}\right\}\right] \equiv$$

$$\equiv \frac{1}{\varepsilon(\mu_2 - \mu_1)} \sum_{k=1}^{2} (-1)^k \mu_k \exp\left\{\mu_k(\rho - R) + \mu_k^2 \mathrm{Fo}\right\} \mathrm{erfc}\left\{\mu_k \sqrt{\mathrm{Fo}} + \frac{\rho - R}{2\sqrt{\mathrm{Fo}}}\right\}$$

for $\mu_k = (2\varepsilon)^{-1}[R^2 + (-1)^k \sqrt{R^4 - 4\varepsilon(\mathrm{Bi} + R)}]$, $k \cup \{1, 2\}$,

$$\left(\mathrm{Bi} = \frac{R^4}{4\varepsilon} - R\right) \Rightarrow \varphi(\rho, \mathrm{Fo}) = \frac{1}{\varepsilon} L^{-1}\left[\frac{\exp\left\{-(\rho - R)\sqrt{s}\right\}}{(\sqrt{s} + \mu)^2}\right] \equiv$$

$$\equiv \frac{1}{\mu^2} \mathrm{erfc}\left\{\frac{\rho - R}{2\sqrt{\mathrm{Fo}}}\right\} - \frac{2}{\mu}\sqrt{\frac{\mathrm{Fo}}{\pi}} \exp\left\{-\frac{(\rho - R)^2}{4\mathrm{Fo}}\right\} +$$

$$+\left(2\mathrm{Fo} + \frac{\rho - R}{\mu} - \frac{1}{\mu^2}\right)\exp\left\{\mu(\rho - R) + \mu^2 \mathrm{Fo}\right\} \mathrm{erfc}\left\{\frac{\rho - R}{2\sqrt{\mathrm{Fo}}} + 2\sqrt{\mathrm{Fo}}\right\} \quad (14)$$

for $\mu = (2\varepsilon)^{-1} R^2$, and

$$\left(\mathrm{Bi} > \frac{R^4}{4\varepsilon} - R\right) \Rightarrow \varphi(\rho, \mathrm{Fo}) = \frac{1}{\varepsilon(\mu_2 - \mu_1)} L^{-1}\left[\frac{1}{\sqrt{s} + \mu_1} - \frac{1}{\sqrt{s} + \mu_2}\right] \equiv$$

$$\equiv i(2\varepsilon\gamma) \sum_{k=1}^{2} (-1)^{k-1} \mu_k \exp\left\{\mu_k(\rho - R) + \mu_k^2 \mathrm{Fo}\right\} \mathrm{erfc}\left\{\mu_k \sqrt{\mathrm{Fo}} + \frac{\rho - R}{2\sqrt{\mathrm{Fo}}}\right\},$$

where

$$\mu_k = (2\varepsilon)^{-1}\left[R^2 + i(-1)^k \sqrt{4\varepsilon(\mathrm{Bi} + R) - R^4}\right], \quad k \in \{1, 2\};$$

$$\gamma = (2\varepsilon)^{-1} \sqrt{4\varepsilon(\mathrm{Bi} + R) - R^4};$$

$$\mathrm{erfc}\{\xi + i\eta\} = \mathrm{erfc}\{\xi\} - \frac{2}{\sqrt{\pi}} \exp\left\{-\xi^2\right\} \int_0^{\eta} \exp\left\{y^2\right\} [\sin(2\xi y) + i\cos(2\xi y)]\, dy$$

is the complementary Gaussian error function of the imaginary argument and

$$\mathrm{erfc}\{\xi\} = \frac{2}{\sqrt{\pi}} \int_{\xi}^{\infty} \exp\left\{-y^2\right\} dy$$

is the complementary Gaussian error function of the real argument [2].

Investigations of the temperature state of the surface $\rho = R$ of the spherical heating source are of greatest practical interest. This is important in both evaluating the maximum possible temperatures in the solid body and in establishing the possibility of acting on its temperature field in a controlled manner. We consider these problems in greater detail, restricting further analysis to the case $\zeta(\mathrm{Fo}) = 1$, i.e., taking the temperature of the gas filling the spherical cavity to be constant. Along with the available physical interpretation [11, 13], this case is important in testing the results obtained, since it leads to the simplest representations of the solution $\Theta(\rho, \mathrm{Fo})$ of problem (2).

Let $\tilde{\Theta}(R, \mathrm{Fo})$ be the temperature of the surface $\rho = R$ of a solid body containing a spherical heating source with an inert thermally thin coating of its surface. For the function $\Theta(\rho, \mathrm{Fo})$, representation (12) in this case takes the form

$$\Theta(\rho, \mathrm{Fo}) = \tilde{\Theta}(\rho, \mathrm{Fo}) - \int_0^{\mathrm{Fo}} Q(\mathrm{Fo} - \tau)\, \varphi(\rho, \tau)\, d\tau, \quad \rho \ge R, \quad \mathrm{Fo} \ge 0, \tag{15}$$

where the function $\varphi(\rho, \mathrm{Fo})$ is determined by equalities (14) and the function $\tilde{\Theta}(R, \mathrm{Fo})$ for $\zeta(\mathrm{Fo}) = 1$ will be written, according to (12) and (13), as

$$\tilde{\Theta}(R, \mathrm{Fo}) = \frac{\mathrm{Bi}}{\varepsilon} L^{-1}\left[\frac{1}{s\left\{s + \varepsilon^{-1}(R^2\sqrt{s} + \mathrm{Bi} + R)\right\}}\right]. \tag{16}$$

The corresponding variants of representation (16) of the function $\Theta(\tilde{R}, \mathrm{Fo})$ depending on the character of the roots of the quadratic equation $z^2 + \varepsilon^{-1}\,[R^2 z + (\mathrm{Bi} + R)] = 0$ have the form [2, 19]

$$\left(\mathrm{Bi} < \frac{R^4}{4\varepsilon} - R\right) \Rightarrow \tilde{\Theta}(R, \mathrm{Fo}) = \frac{\mathrm{Bi}}{\varepsilon(\mu_2 - \mu_1)} L^{-1}\left[\frac{1}{(s(\sqrt{s} + \mu_1)} - \frac{1}{s(\sqrt{s} + \mu_2)}\right] \equiv$$

$$\equiv \frac{\mathrm{Bi}}{\mathrm{Bi} + R}\left[1 - \frac{\mu_2 - \mu_1}{\mu_1 \mu_2} \sum_{k=1}^{2} \frac{(-1)^k}{\mu_k} \exp\left\{\mu_k^2 \mathrm{Fo}\right\} \mathrm{erfc}\left\{\mu_k \sqrt{\mathrm{Fo}}\right\}\right]$$

for $\mu_k = (2\varepsilon)^{-1}[R^2 + (-1)^k \sqrt{R^4 - 4\varepsilon(\mathrm{Bi} + R)}]$, $k \cup \{1, 2\}$,

$$\left(\mathrm{Bi} = \frac{R^4}{4\varepsilon} - R\right) \Rightarrow \tilde{\Theta}(R, \mathrm{Fo}) = \frac{\mathrm{Bi}}{\varepsilon} L^{-1}\left[\frac{1}{(s(\sqrt{s} + \mu)^2}\right] \equiv$$

$$\equiv \frac{\mathrm{Bi}}{\varepsilon\mu^2}\left[1 - 2\mu\sqrt{\frac{\mathrm{Fo}}{\pi}} - (1 - 2\mu^2 \mathrm{Fo}) \exp\left\{\mu^2 \mathrm{Fo}\right\} \mathrm{erfc}\left\{\mu\sqrt{\mathrm{Fo}}\right\}\right] \tag{17}$$

for $\mu = (2\varepsilon)^{-1} R^2$, and

$$\left(\mathrm{Bi} > \frac{R^4}{4\varepsilon} - R\right) \Rightarrow \tilde{\Theta}(R, \mathrm{Fo}) = \frac{\mathrm{Bi}}{\varepsilon(\mu_2 - \mu_1)} L^{-1}\left[\frac{1}{(s(\sqrt{s} + \mu_1)} - \frac{1}{s(\sqrt{s} + \mu_2)}\right] \equiv$$

$$\equiv \frac{\mathrm{Bi}}{\mathrm{Bi} + R}\left[1 - \frac{1}{\gamma} \mathrm{Im}\left(\bar{\mu}_1 \exp\left\{\mu_1^2 \mathrm{Fo}\right\} \mathrm{erfc}\left\{\mu_1 \sqrt{\mathrm{Fo}}\right\}\right)\right] =$$

$$= \frac{\mathrm{Bi}}{\mathrm{Bi} + R}\left[1 - \frac{1}{\gamma}\left\{\frac{2}{\sqrt{\pi}}[\beta \cos(2\beta\gamma \mathrm{Fo}) + \gamma \sin(2\beta\gamma \mathrm{Fo})]\right\} \exp\left(-\gamma^2 \mathrm{Fo}\right) \times\right.$$

$$\times \int_0^{\gamma\sqrt{\mathrm{Fo}}} \exp\left\{y^2\right\} \cos(2\beta\sqrt{\mathrm{Fo}}\, y)\, dy + [\gamma \cos(2\beta\gamma \mathrm{Fo}) - \gamma \sin(2\beta\gamma \mathrm{Fo})] \times$$

$$\left.\times \exp\left\{(\beta^2 - \gamma^2)\mathrm{Fo}\right\}\left(\mathrm{erfc}\left\{\beta\sqrt{\mathrm{Fo}}\right\} - \frac{2}{\sqrt{\pi}} \exp\left\{-\beta^2 \mathrm{Fo}\right\} \int_0^{\gamma\sqrt{\mathrm{Fo}}} \exp\left\{y^2\right\} \sin(2\beta\sqrt{\mathrm{Fo}}\, y)\, dy\right)\right],$$

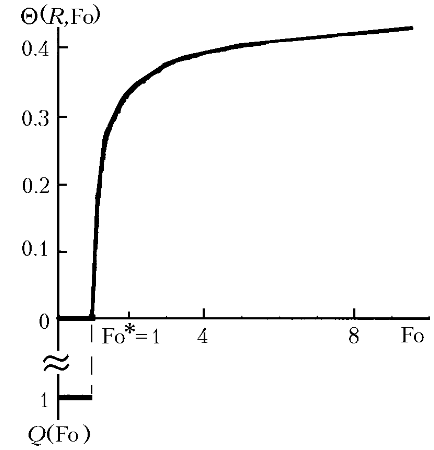

Fig. 2. Temperature profile Θ(R, Fo) of a solid body with a spherical heating source in realization of a pulsed regime of heat absorption in a thermally thin coating.

where $\bar{\mu}_1 = \mu_2$, $\mu_k = (2\varepsilon)^{-1}\ [R^2 + i(-1)^k \sqrt{4\varepsilon(\mathrm{Bi} + R) - R^4}]$, $k \in \{1, 2\}$, $\beta = (2\varepsilon)^{-1}R^2$, and $\gamma = (2\varepsilon)^{-1}\sqrt{4\varepsilon(\mathrm{Bi} + R) - R^4}$.

Analysis of the Results Obtained. When $Q(\mathrm{Fo}) = 0$, equalities (15) and (17) determine the temperature profile of the surface $\rho = R$ of the infinite solid body containing a spherical heating source with an inert thermally thin coating at a constant temperature of the gas ($\zeta(\mathrm{Fo}) = 1$). It can be shown that, for high Fo values, equality (17) leads to the following asymptotic estimate of the maximum temperature of the surface $\rho = R$ of the spherical source:

$$\Theta\ (R, \mathrm{Fo}) \sim \frac{\mathrm{Bi}}{\mathrm{Bi} + R}\left[1 - \frac{R^2}{(\mathrm{Bi} + R)\ \sqrt{\pi \mathrm{Fo}}}\right].$$

It follows that the qualitative character of behavior of the function Θ(R, Fo) is dependent on both the thickness of the thermally thin coating and the intensity of heat transfer on its surface for $\mathrm{Fo} \to +\infty$, i.e., $\Theta(R, \mathrm{Fo}) \to \mathrm{Bi}/(\mathrm{Bi} + R)$ for $\mathrm{Fo} \to +\infty$. Thus, the presence of the thermally thin coating leads to a reduction in the value of the maximum attainable heating of the solid-body surface, which improves its thermal protection. At the same time, an analogous evaluation of the maximum temperature of a half-space with a thermally thin coating of its surface in high-temperature heating by the ambient medium leads us to a fundamentally different conclusion [17]: the value of the maximum attainable heating is dependent on neither the thickness of the coating nor the intensity of heat transfer on its surface in this case.

We consider the case $Q(\mathrm{Fo}) > 0$ without specifying the form of the function $Q(\mathrm{Fo})$ characterizing the realized law of heat absorption in a thermally thin coating. For preliminary analysis of the possibility of acting on the temperature field of a solid body in a controlled manner, we direct our attention to equality (12). Theoretically we can always find the law of heat absorption $Q(\mathrm{Fo})$ in the thermally thin coating of a spherical heating source for a prescribed law $\zeta(\mathrm{Fo})$ of variation in the source temperature. Thus, e.g., if $\Theta(R, \mathrm{Fo}) = 0 = \mathrm{const}$ (thermostating regime), then, according to (12), we obtain

$$Q\ (\mathrm{Fo}) = \mathrm{Bi}\ \zeta\ (\mathrm{Fo})\ . \tag{18}$$

When $\zeta(\mathrm{Fo}) = 1$, it follows from (18) that $Q(\mathrm{Fo}) = \mathrm{Bi} = \mathrm{const}$, i.e., the law of heat absorption in the thermally thin coating is uniquely determined by the intensity of heat transfer on its surface $\rho = 1$. The thermal inertia of the coating, unlike [20], exerts no influence on the temperature profile of the heat-insulated surface of the solid body.

Thermoactive coatings with a time-variable specific power of heat absorption in a thermally thin layer are of practical interest. As an example illustrating the possibility of acting, in a controlled manner, on the temperature field of a solid body with the use of coatings of this kind, we consider a pulsed regime with a constant specific power of heat absorption in the coating Q_0:

$$Q\,(\mathrm{Fo}) = Q_0\,[\eta\,(\mathrm{Fo}) - \eta\,(\mathrm{Fo} - \mathrm{Fo}^*)]\;.$$

Figure 2 gives the temperature profile of the surface $\rho = R$ of the solid body with a spherical heating source at a constant temperature of the gas contained in it ($\zeta(\mathrm{Fo}) = 1$). The calculation has been carried out for $\mathrm{Bi} = 1 = Q_0$, the duration of the phase of heat absorption in the coating $\mathrm{Fo}^* = 1$, and $\varepsilon \to +0$. It is seen that the "compensating" heat absorption in the thermally thin coating enables us to thermostat the surface of the solid body during a prescribed time interval $\mathrm{Fo} \cup [0, \mathrm{Fo}^*)$, maintaining its surface temperature constant. Also, it is noteworthy that the qualitative character of behavior of the function $\Theta(R, \mathrm{Fo})$ for high Fo values is the same as that for $Q(\mathrm{Fo}) = 0$: $\Theta(R \to 1 + 0, \mathrm{Fo}) \to 0.5$ for $\mathrm{Fo} \to +\infty$.

Thus, the results of the investigations carried out point to the possibility of acting, in a controlled manner, on the temperature field of an infinite solid body containing a spherical heating source with a thermoactive thermally thin coating of its surface, which ensures programmed variation in the temperature of the heat-insulated surface. Further prospects for acting on the temperature field of a solid body with a spherical heating source in a controlled manner may also involve the realization of nonstationary heat-exchange regimes associated with destruction of the surface layers of the thermoactive coating in the process of its high-temperature heating [21–23] in the system under study [8].

NOTATION

a, thermal diffusivity, m/sec^2; Bi, Biot number; C, set of complex numbers; Fo, Fourier number; Fo^*, duration of the phase of heat absorption in the thermally thin coating; *f*, dimensionless specific power of heat absorption in the coating; *i*, imaginary unit; $L[\bullet]$, operator of the direct integral Laplace transformation; $L^{-1}[\bullet]$, operator of the inverse integral Laplace transformation; $L^2[R, +\infty)$, linear space of functions square-integrable on a semiinfinite interval $[R, +\infty)$; *Q*, integral quantity characterizing the realized regime of heat absorption in the thermally thin coating; *q*, specific (per unit volume) power of heat absorption in the coating, W/m^3; *R*, dimensionless radius of the contact surface of the solid body with the coating; *r*, radius, m; r_1, radius of the spherical heating source, m; r_2, radius of the contact surface of the solid body with the coating, m; *s*, parameter of the integral Laplace transformation; *T*, temperature, K; *t*, time, sec; α, heat-transfer coefficient, W/(m^3·K); ε, dimensionless governing parameter of the "lumped-capacitance" model; ζ, dimensionless temperature of the ambient medium; $\eta(\bullet)$, Heaviside unit function; Θ, dimensionless temperature; $\langle\Theta\rangle$, mean-integral temperature; Λ, dimensionless parameter characterizing the relative thermal conductivity of the solid body; λ, thermal conductivity, W/(m^3·K); ρ, dimensionless radius; χ, dimensionless parameter characterizing the thermoinertial properties of the solid body relative to the coating. Subscripts: c, coating; m, ambient medium; 0, initial value; 1, coating surface subjected to high-temperature heating; 2, contact surface of the solid body with the coating.

REFERENCES

1. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids* [Russian translation], Nauka, Moscow (1964).
2. A. V. Luikov, *Heat-Conduction Theory* [in Russian], Vysshaya Shkola, Moscow (1967).
3. E. M. Kartashov, *Analytical Methods in the Theory of Heat Conduction of Solids* [in Russian], Vysshaya Shkola, Moscow (2001).
4. H. Parkus, *Instantionäre Wärmespannungen* [Russian translation], Fizmatgiz, Moscow (1963).
5. A. G. Merzhanov and F. I. Dubovistkii, State-of-the-art of the theory of thermal explosion, *Usp. Khim.*, **35**, No. 4, 656–683 (1966).
6. D. A. Frank-Kamenetskii, *Diffusion and Heat Transfer in Chemical Kinetics* [in Russian], Nauka, Moscow (1987).
7. E. M. Kartashov, V. M. Nechaev, and G. M. Bartenev, Thermoelastic reaction of an infinite medium with a spherical heated cavity, *Fiz. Khim. Obrab. Mater.*, No. 2, 26–35 (1981).
8. A. G. Merzhanov, V. V. Barzykin, and V. G. Abramov, The theory of thermal explosion: From N. N. Semenov to our days, *Khim. Fiz.*, **15**, No. 6, 3–44 (1966).

9. A. V. Attetkov and I. K. Volkov, A temperature field of the region with a spherical heating source, *Vestn. MGU im. N. É. Baumana*, *Estestvennye Nauki*, No. 1, 42–50 (2001).
10. G. A. Nesenenko, *Solution of Problems of Irregular Nonlinear Heat and Mass Transfer by a "Geometrical-Optical" Asymptotic Method* [in Russian], Izd. Birsk. GPI, Birsk (2003).
11. B. A. Khasainov, A. V. Attetkov, and A. A. Borisov, Shock-wave initiation of energy materials and a viscous-plastic model of hot spots, *Khim. Fiz.*, **15**, No. 7, 55–125 (1996).
12. G. I. Kanel', S. V. Razorenov, A. V. Utkin, and V. E. Fortov, *Shock-Wave Phenomena in Condensed Media* [in Russian], Yanus-K, Moscow (1996).
13. L. P. Orlenko, *Physics of Explosion* [in Russian], in 2 vols., Vol. 1, Fizmatlit, Moscow (2002).
14. S. G. Andreev, Influence of nonexplosive chemical additives on decomposition of energy materials under low-velocity mechanical and shock-wave effects, *Khim. Fiz.*, **19**, No. 2, 76–81 (2000).
15. V. P. Il'in, S. P. Smirnov, and A. S. Smirnov, Correlation between the critical parameters of excitation of explosion of mixture explosives and their composition and structure. Principles of compounding low-sensitive compositions, in: *Ext. Abstr. of Papers Presented at Int. Conf. "III Kharitonov Topical Scientific Readings"* [in Russian], Sarov (2001), pp. 7–8.
16. M. A. Pudovkin and I. K. Volkov, *Boundary-Value Problems of the Mathematical Theory of Heat Conduction as Applied to Calculations of Temperature Fields in Oil Beds under Flooding* [in Russian], Izd. Kazansk. Univ., Kazan' (1978).
17. A. V. Attetkov, P. A. Vlasov, and I. K. Volkov, Temperature field of a half-space with a thermally thin coating in pulsed modes of heat exchange with the environment, *Inzh.-Fiz. Zh.*, **74**, No. 3, 81–86 (2001).
18. O. A. Ladyzhenskaya, V. A. Solonnikov, and N. N. Ural'tseva, *Linear and Quasilinear Parabolic Equations* [in Russian], Nauka, Moscow (1967).
19. V. A. Ditkin and A. V. Prudnikov, *Handbook of Operational Calculus* [in Russian], Vysshaya Shkola, Moscow (1965).
20. A. V. Attetkov, I. K. Volkov, and E. S. Tverskaya, Thermoactive lining as a means for controlled effect on the temperature field of the structure, *Izv. Ross. Akad. Nauk*, *Énergetika*, No. 4, 131–141 (2002).
21. G. N. Tret'yachenko and L. N. Gracheva, *Thermal Deformation of Nonmetallic Destructive Materials* [in Russian], Naukova Dumka, Kiev (1983).
22. Yu. I. Dimitrienko, *Mechanics of Composite Materials at High Temperatures* [in Russian], Mashinostroenie, Moscow (1997).
23. É. M. Kartashov, B. Tsoi, and V. V. Shevelev, *Structural-Statistical Kinetics of Polymer Destruction* [in Russian], Khimiya, Moscow (2002).